**Math modeling unit and activity** –**Conditional Probability**

**Activity name: Memphis-Louisville Basketball Game**

Big Idea(s)/ Concept(s)/major math area(s):

**Probability Rules:** (Addition Rule, Multiplication Rule, Conditional Probability)

**Vocabulary:** Independence

Grade level(s): Grade 11 DP IB Math Standard Level/Math Studies

**Math Standards included:** **Standard Level Math:** Descriptive statistics: collection of raw data; display of data in pictorial and diagrammatic forms, including pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs. Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range. Calculating probabilities of simple events.

**Math Studies:** Sample space; event A; complementary event, A′ . Probability of an event. Probability of a complementary event. Expected value.

**Procedure overview/teacher directions:**

This modeling activity involves a basketball game between Louisville and Memphis State in the 2005 NCAA tournament.

The score is 75 to 73 with Louisville ahead of Memphis State.

Louisville is shooting a free throw with three seconds remaining in the game. After they miss the free throw, Memphis brings the ball down the court and attempts a three-point shot but they are fouled. The clock is now at zero and the Memphis player will shoot three free throws at one point for each shot he makes.

The students will decide (using probability calculations and their own assumptions) whether is was wise to foul the shooter. Did the foul give Louisville a better chance of winning or a worse chance?

**Before you begin:** Students should know what it means for events to be independent, and should know how to use empirical probability, the addition rule and the multiplication rule to calculate the compound probability of independent events. For example, if a player has a 72% free throw percentage, they should be able to explain why the player’s probability of making three consecutive free throws is 0.37. An understanding of conditional probabilities is also helpful but should be discovered in the modeling activity.

Students should also have a working knowledge of basketball so they know that a free throw is worth one point and the shooter will get three free throw shots since he was in the act of shooting a three-point shot.

**Note: This activity is part of an overall unit of modeling activities for probability. If the teacher uses the entire modeling unit, the students should have an easier time with the modeling process.**

**Lesson details:**

1. Go to the following website**:** [**http://www.mathalicious.com/lessons/three-shots**](http://www.mathalicious.com/lessons/three-shots)
2. Go the **TEACHER** tab and select **P**
3. Now you should take a few minutes to white board the scenario and some basic rules of basketball that relate. Leave this white board up so everyone has the basic knowledge they need to participate.
4. After this step, put the students in groups. They should use white boards to state their assumptions. Your goal should be to keep the questions as open as possible and inspire the students to consider the question, the information needed and how to develop a model to answer the question.

To Students:

***Should the player have fouled or not? State any assumptions you make and justify your decision.***

Leading questions (If Necessary):

***How many points are needed for Memphis to Win/Tie/Lose?***

***What are the possible outcomes?***

***What is the probability of each outcome?***

***Based on this, determine the probability of Louisville winning if they foul or if they don’t foul?***

***It is possible some students***

At this point, they should spend time in their groups developing a model to answer the solution. They should prepare a white board to present their model and answer the question. Graphs, equations, data tables and drawings are excellent models.

After they present their model, they should take a moment to reflect on what they have discovered and learned so far.

1. Watch the first shot.

***What outcomes remain?***

***What is the new probability of Louisville Winning/Losing/tying?***

1. Watch the second shot.

***What outcomes remain?***

***What is the new probability of Louisville Winning/Losing/tieing?***

1. Discuss probability notation and how to use the notation to set up probability problems involving the Addition Rule, Multiplication Rule and Conditional Probability.
2. Give the statistics for free throw percentage and three-point percentage. Have them recalculate the probabilities and put them into a table for analysis.
3. ***What is the likelihood of winning given that the foul is committed and given that the foul is not committed? Should you foul?***

At this point, the teacher can give them more time to reflect. Reflection is critical to learning. If they made any mistakes, they should reflect on their mistake. Why did you make a mistake? How did you correct it? Mistakes are when new brain synapses are created and this opportunity should be maximized.

**Follow on discussion:**

* ***Are the shots independent of each other?***
* ***If not, how would that change the decision to foul?***
* ***Should Memphis State have attempted the three point shot or should they have tried to tie the score with a higher percentage two point shot? What was their decision process?***
* ***If Louisville had not-fouled, would they have lost?***

Show the final outcome video. (NEED TO GET THIS OFF OF YOUTUBE)

***The Math:***

F – foul OW – win in overtime (ties then wins in overtime)

N – no foul OL – lose in overtime (ties then loses in overtime)

W – win in regulation m – miss a shot (.28)

L – lose in regulation h – hit a shot (.72)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Lousiville Win | Louisville Lose | Overtime Win | Overtime Lose |
| Foul | Condition | Makes 0 or 1 | Makes 3 | Makes 2/OT Win | Makes 2/OT Loss |
| Probability | P(W|F) | P(L|F) | P(OW|F) | P(OL|F) |
| No Foul | Condition | Misses 3PT shot | Makes 3PT shot |  |  |
| Probability | P(W|N) | P(L|N) | P(OW|N) | P(OL|N) |

$$P\left(F\right)=P\left(mmm\right)+P\left(hmm\right)+P\left(mhm\right)+P\left(mmh\right)=P\left(mmm\right)+3P\left(hmm\right)$$

 $=\left(.28\right)\left(.28\right)\left(.28\right)+3\left(.72\right)\left(.28\right)\left(.28\right)=.1913$

$P\left(F\right)=P\left(hhh\right)$ $=\left(.72\right)\left(.72\right)\left(.72\right)=.3732$

$$P\left(F\right)=\left\{P\left(mhh\right)+P\left(hhm\right)+P\left(hmh\right)\right\}P(OW)=3P\left(hhm\right)P(OW)$$

 $=3\left[\left(.72\right)\left(.72\right)(.28)\right]\left(.5\right)=.2177$

$$P\left(F\right)=\left\{P\left(mhh\right)+P\left(hhm\right)+P\left(hmh\right)\right\}P(OL)=3P\left(hhm\right)P(OL)$$

 $=3\left[\left(.72\right)\left(.72\right)\left(.28\right)\right](.5)=.2177$

$$P\left(N\right)=.6$$

$$P\left(N\right)=.4$$

$$P\left(N\right)=P\left(N\right)=0$$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Lousiville Win | Louisville Lose | Overtime Win | Overtime Lose |
| Foul | Condition | Makes 0 or 1 | Makes 3 | Makes 2/OT Win | Makes 2/OT Loss |
| Probability | .19 | .37 | .22 | .22 |
| No Foul | Condition | Misses 3PT shot | Makes 3PT shot |  |  |
| Probability | .60 | .40 | 0 | 0 |

$P\left(F\right)=.19+.22$=.41

$P\left(N\right)=.$6